length of the filter. Investigation of (23) shows that the reflection coefficient is roughly proportional to the inverse square of the length l of the filter. Thus, reflection level can always be reduced to meet a particular prescription by increasing the length l. Accordingly, we can say that increasing n larger than three in raised cosine exponential should increase the reflection coefficient. This is confirmed as shown in Fig. 4. We conclude, therefore, that the exponential function raised to cosine cube is the best in a frequency band where there is possible mode conversion, whereas the exponential function raised to cosine square is the best in a frequency band where there is no possible mode conversion.

Finally we compare the reflection characteristics of a symmetrical whole filter with that of a nonsymmetrical half-section filter. The reflection characteristics of a half-section filter can be obtained also from (23) by replacing the upper limit l by zero. This is done for (23C) for delta impedance variation with

$$\left(1 - \frac{\lambda_p^2}{a_1^2}\right) = 0.005$$

and is also plotted in Fig. 3 for comparison with the symmetrical whole section. It is interesting to note that the reflection of the symmetrical filter never exceeds that of the nonsymmetrical filter section by 6 db, which indicates that the reflections of the two half-sections just add in phase. Of course, the peaks of the reflection characteristics of the symmetrical filter represent the complete cancellations. We also see that the steepness of the reflection characteristics for the symmetrical whole section is much greater than that of the nonsymmetrical half-section as shown in Fig. 3.

Conclusions

In designing a tapered waveguide high-pass filter, it is found that the logarithmic derivative of the impedance variations along the taper for a frequency near the cutoff can yield the most needed information in determining the mechanical profile of a filter that will meet the prescribed requirements. Among many illustrated simple trial functions of impedance variations along the tapered filter, the exponential function raised to cosine square gives reflection characteristics with the steepest rise near the cutoff and the lowest reflection for all frequencies beyond the cutoff. The steep rise of the reflection characteristics near the cutoff is phenomenal, since, for example, at the nominal cutoff of 55 kMc the reflection reduces to about −50 db within 0.18 kMc.

Finally we should note that the same design procedure for the high-pass filter can be used for waveguide transitions of extremely wide band and very low reflections.

Acknowledgment

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Matching into Band-Pass Transmission Structures

M. L. HENSEL, MEMBER, IEEE, AND E. O. SCHULZ-DuBOIS

Summary—**This paper describes a method for broad banding the matching transition from a low-dispersion transmission line to a high-dispersion iterated filter structure. A good match can be obtained over essentially the entire pass band of the filter structure. To accomplish this the band at the end of the structure is widened beyond both nominal cutoff frequencies. It is narrowed down to the regular structure bandwidth in a taper extending over a few filter elements. In the comb structure used for traveling wave masers, a return loss of 20 db (VSWR = 1.2) or better is realized over 90 per cent of the pass band with a taper including four comb fingers. Several examples of suitable taper designs are given. Each of these, however, requires empirical adjustment in order to produce an optimum match.**

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Introduction

THE MATCHING transition from a coaxial transmission line to the comb-type slow-wave structure as used in traveling wave masers[1,2] shows a number of features which may be considered typical for the matching situation encountered with other band-pass filter structures. 1) Generally the shape of the matching element is derived empirically. Here it is a preshaped wire as shown in Fig. 1(a). The extra inductance introduced by the loop in the wire is essential in this matching scheme. 2) A transition to a uniform slow-wave

[1] R. W. DeGrasse, E. O. Schulz-DuBois and H. E. D. Scovil, *Bell Syst. Tech. J.*, "The three-level solid state traveling wave-maser," vol. 38, pp. 305–334; March, 1959.

[2] M. L. Hensel and E. B. Treacy (to be published).

structure consisting of a single element may have little or no dispersion. Thus it can produce a satisfactory match only over a limited frequency range. The reflectometer tracing of a typical comb structure match [Fig. 1(b)] may illustrate this point. Very low reflection is observed over a very small fraction of the structure bandwidth only and the quality of the match deteriorates for frequencies further away. For many applications, this match may be satisfactory over about one third of the structure pass band. 3) Usually some adjustable feature is incorporated in the matching element. Here the distance of the wire probe to the first comb finger may be changed by a movable shorting plate. As illustrated in Fig. 1(b), this moves the frequency of the best match within the structure pass band.

This situation is in agreement with theoretical arguments. The comb-type slow-wave structure and other band-pass filter structures are usually designed for a small fractional transmission band. In an appropriate lumped element equivalent circuit, their impedance is highly dispersive and in the comb, for example, assumes the values zero and infinity at the band edges. By contrast, the connecting transmission lines such as waveguides, coaxials or other TEM lines have an impedance of low or zero dispersion. It is well known[3] that a simple transformer (*e.g.*, a quarter wave transformer in the case of real impedances) can always be designed to connect without reflection two transmission lines which have arbitrary but specified impedances. This means that a perfect match can always be achieved at any particular single frequency within the pass band. Raub's quasi-static method[4] may be used to find a suitable geometry for the transformer on an analytical basis. As an alternative, the shape can be derived empirically. In either case some adjustment may be desirable for tuning and controlling the resulting match. It is clear, however, that this approach should lead to a narrow-band match only. The impedance ratio varies rapidly with frequency, whereas the transformer is usually able to compensate for one particular ratio only. This argument assumes, of course, that the matching transformer itself does not show high-dispersion.

A broad-band match apparently requires some high dispersion in the matching device itself so that appropriate impedance ratios can be realized at two or more frequencies within the pass band. These additional impedance conditions can usually be satisfied only in a more extended matching network which may contain one or more resonating circuits similar to the ones used in the filter structure itself. Brillouin[5] has derived a criterion which may shed some light on this problem and which may be reformulated as follows. If, in an iterated filter structure, there is direct coupling not just between adjacent filter elements but between N-nearest neighbors, then it is a condition for perfect broad-band impedance matching that the matching structure produce a voltage and current distribution on the first element of the structure equal to that produced normally by N-neighboring resonating elements. In the comb structure there is coupling of this type where, in practice, N may perhaps be considered as limited to four or five. This criterion indicates the need for a multielement matching structure.

In the remainder of the paper a matching technique is described which satisfies these premises. It is tolerant in the sense that it does not require the use of certain well-defined impedance ratios but rather can be made to work by a coarse arrangement of impedances in ascending or descending sequences. It is empirical in the sense that a successful design can only be derived by experimentation and empirical adjustment for minimum return loss.

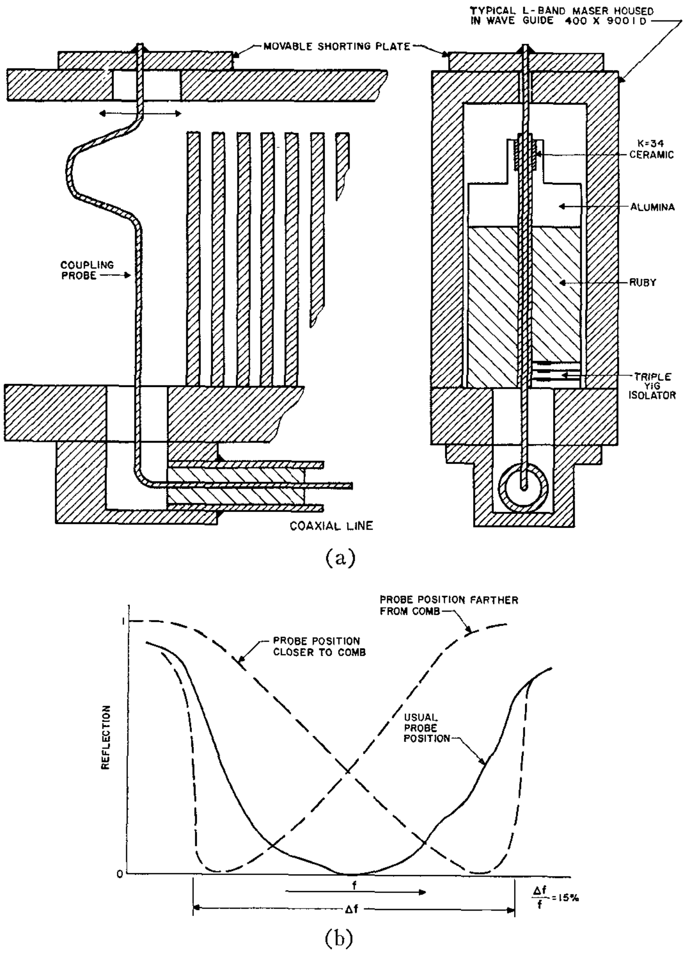

Fig. 1—Conventional match between coaxial line and maser comb structure. (a) Design of the probe coupler. (b) Typical reflectomer tracings obtained.

[3] R. E. Collin, "Field Theory of Guided Waves," McGraw-Hill Book Company, Inc., New York, N. Y.; 1960. See especially p. 383.

[4] Wayne E. Raub, "The Use of Quasi-Static Mode Approximations in the Design of Slow-Wave Structure Impedance Matches," Paper No. 27/3, presented at 1961 IRE Wescon Conv.

[5] Léon Brillouin, "Wave Propagation in Periodic Structures," Dover Publications, New York, N. Y.; 1953. See p. 28. This reference was pointed out to us by E. B. Treacy.

Principle of the Broad-Band Match

The matching section consists of a matching probe and a few elements of the iterated filter structure. The matching probe may be that shown in Fig. 1(a) and in connection with a uniform filter structure it would produce a good match at only one frequency near midband as shown in Fig. 1(b). The resonant elements of the filter structure belonging to the matching transition may be called the taper section. In this section, as illustrated in Fig. 2(a), the frequency width of the slow-wave structure is gradually increased both at the upper and lower cutoff frequency. The taper may be roughly linear or exponential. The illustration is typical, however, in the sense that the taper extends over about four resonators and that the increase in frequency width is only about 30 per cent at either end. The corresponding impedances are shown in Fig. 2(b). In a lumped circuit analog, the impedance of each element goes from zero to infinity across its pass band. Also shown is the frequency independent impedance of the input coaxial cable. It is assumed that the tapering leaves the midband impedance of all elements approximately unchanged. Then a signal in the lower half of the pass band goes through a sequence of descending impedances as it enters the structure from the input cable. This is indicated by the numbered points of Fig. 2(b). The opposite applies to a signal in the upper half of the pass band. The phase shifts between the excitations of adjacent elements are shown in Fig. 2(c). They vary from zero to π across the pass band. It is assumed that the midband phase shift is substantially unaltered by the tapering. A signal in the lower half of the pass band entering the structure from the input cable experiences a series of descending phase shifts as indicated by the numbered points in Fig. 2(c). The opposite is true for a signal in the upper half of the pass band.

From the impedances and phase shifts, the resulting voltage reflection coefficient can be determined as a function of frequency across the band. If the impedance changes only by a small amount at every step in the taper, then interference, *i.e.*, the effects of multiple reflections, can be neglected and, as a consequence, the resulting reflection coefficient is simply the vector sum of the individual complex reflection coefficients. In Fig. 2(d) the vector sum is carried out for a particular frequency. In the sketch, the angles between successive reflection vectors are just twice the phase angles taken from Fig. 2(c). For comparison, the voltage reflection coefficient to be expected in the absence of the matching taper is also shown.

An inspection of the graphs shows that the matching scheme drastically reduces the resulting reflection coefficient. The original ratio between two appreciably different impedance values is broken up into a number of ratios near unity. The phase angles between filter elements in the taper gravitate around 90 degrees. Thus the function of the match may be described as that of a number of generalized quarter-wave transformers in which neither the impedance ratios are exactly equal nor the phase is exactly 90 degrees. The low over-all reflection results from the fact that each incremental transformer contributes a small reflection and that, due to the phase relations, these individual reflections cancel almost completely. The improvement attainable by this matching technique may be estimated by a crude theoretical argument or by inspection of vector diagrams like that of Fig. 2(d). The result is that the voltage reflection present without the taper may be reduced to order $1/N$ of its original value where N is the number of resonators in the taper. Thus a four-element taper could be expected to reduce a VSWR of 2.0 to 1.2 which is in reasonable agreement with experimental results.

Broad-Band Matching into the Maser Comb Structure

Design procedures[6] are known for the dielectrically loaded comb structure which allows the independent control of the upper and lower cutoff frequencies.[7] Instead of discussing means to broaden the pass band it is therefore more convenient to discuss separately design modifications which increase the upper cutoff frequency and others which decrease the lower cutoff frequency. The quantity conveniently varied was the capacitance of the resonant circuit responsible for each cutoff frequency. The capacity was reduced at the upper cutoff frequency and increased at the lower cutoff frequency. With other classes of iterated filter structures it might be more convenient to alter the inductance or a combination of capacitance and inductance.

Near the upper cutoff frequency, the electrical field lines go mostly from finger tip to finger tip extending a little away from the comb. Experimentally the upper cutoff frequency was tapered in the following three ways:

1) The dielectric in immediate contact with the first finger tips is tapered as shown in Fig. 3(a).
2) The width of the end fingers is reduced in a gradual fashion by a lapping operation as shown in Fig. 3(b).
3) The end fingers are spread so that the spacing between finger tips increases as shown in Fig. 3(c).

The first two alternatives appeared less desirable in our L-band maser design because they pose difficult quality control problems. The third method is used in a practical L-band maser design. A ceramic spacer comb is used in these masers[2] to maintain periodic finger spacing in

[6] S. E. Harris, R. W. DeGrasse and E. O. Schulz-DuBois, *Bell Syst. Tech. J.*, vol. 43, pp. 437–484; January, 1964.

[7] A forward wave-comb structure is discussed here. The present matching technique, however, is equally applicable to a backward wave structure.

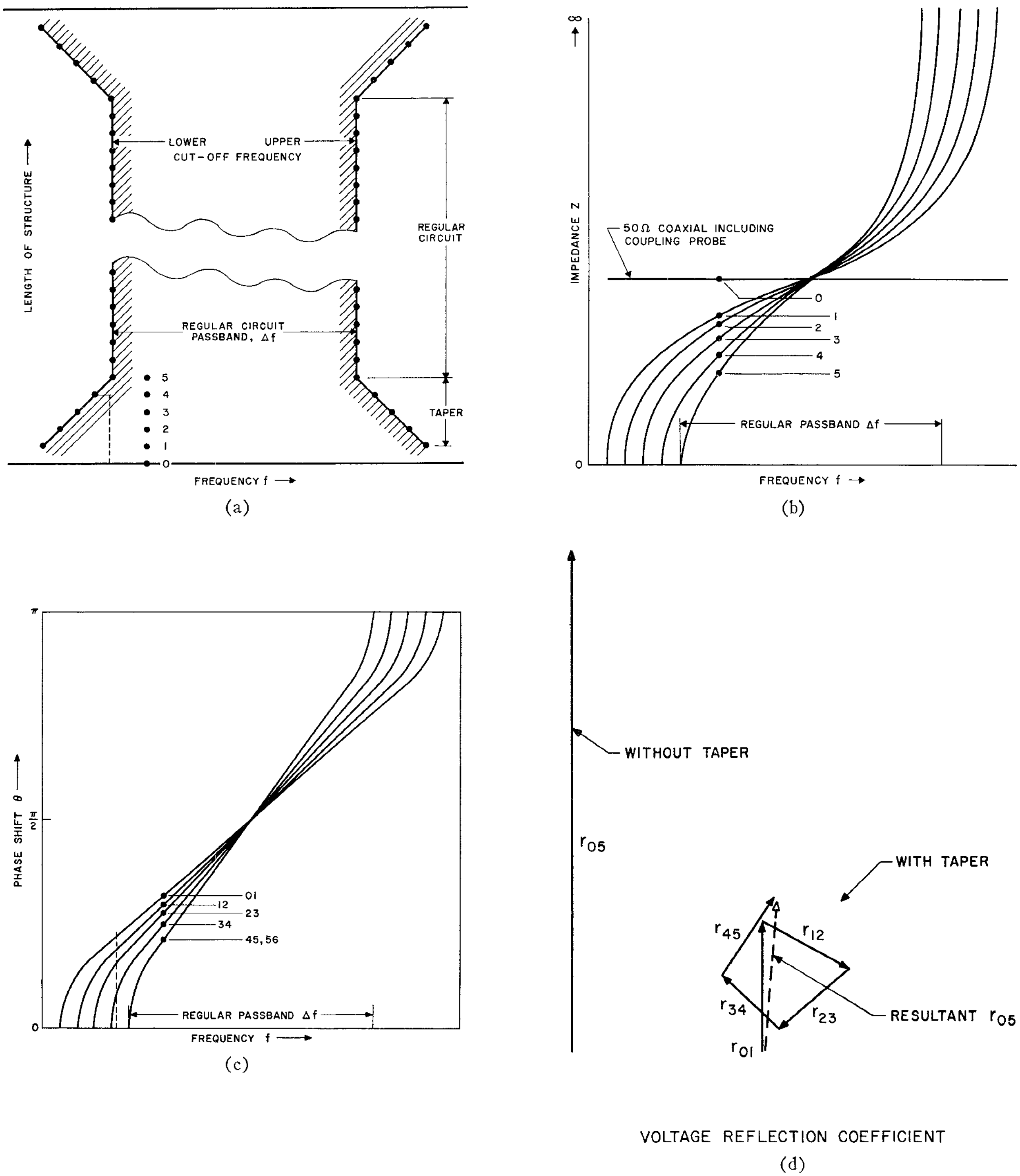

Fig. 2—Illustration of the matching technique based on widening of the pass band. (a) Cutoff frequencies in the matching taper and the slow-wave structure. (b) Impedances as a function of frequency in the matching taper. (c) Phase shift between adjacent filter elements as a function of frequency in the matching taper. (d) Vector diagram of the voltage reflection coefficient with and without taper.

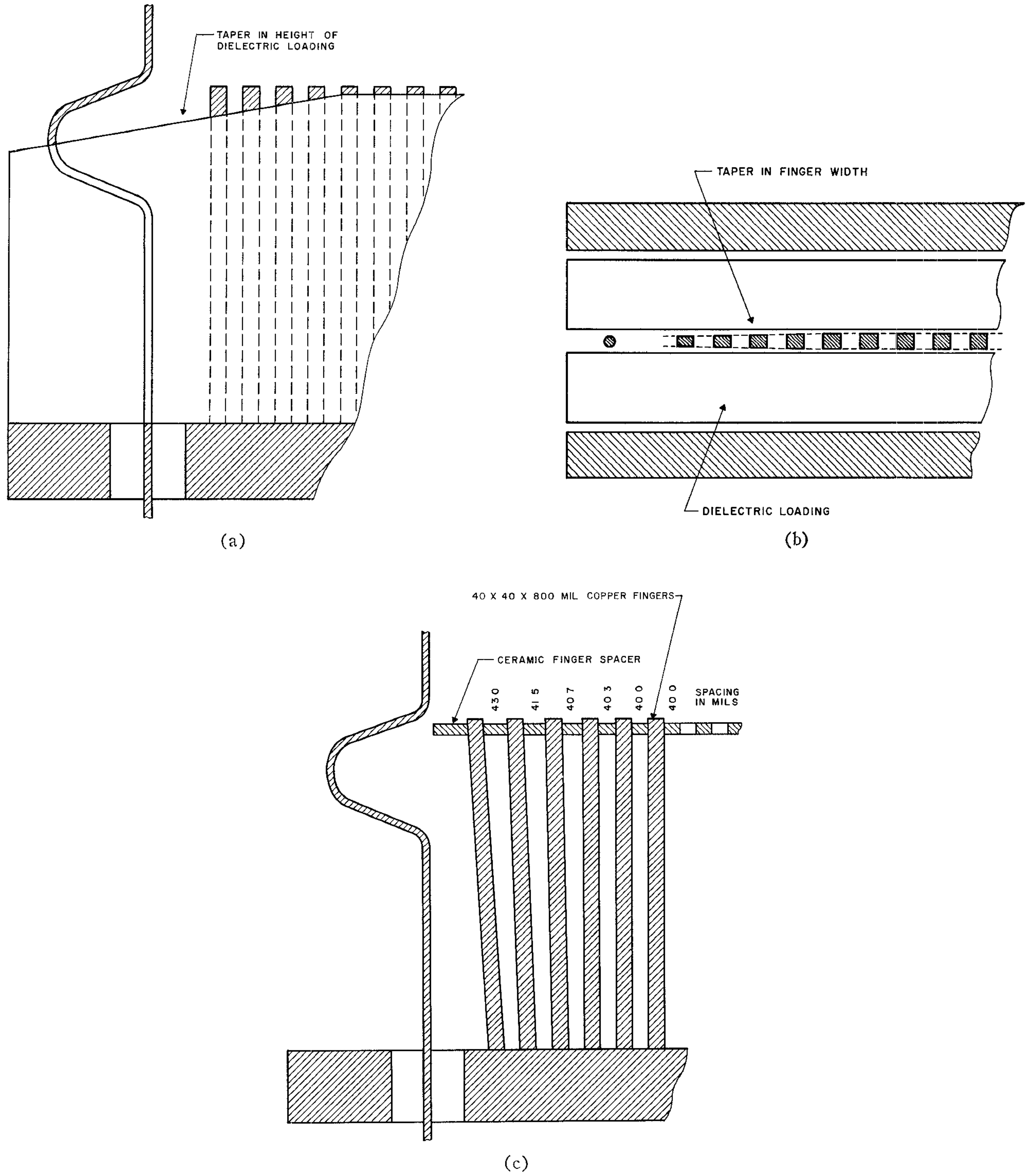

Fig. 3—Methods of increasing the upper cutoff frequency of the maser comb structure. (a) Taper of height of dielectric loading. (b) Taper of comb finger thickness. (c) Taper of spacing between finger tips.

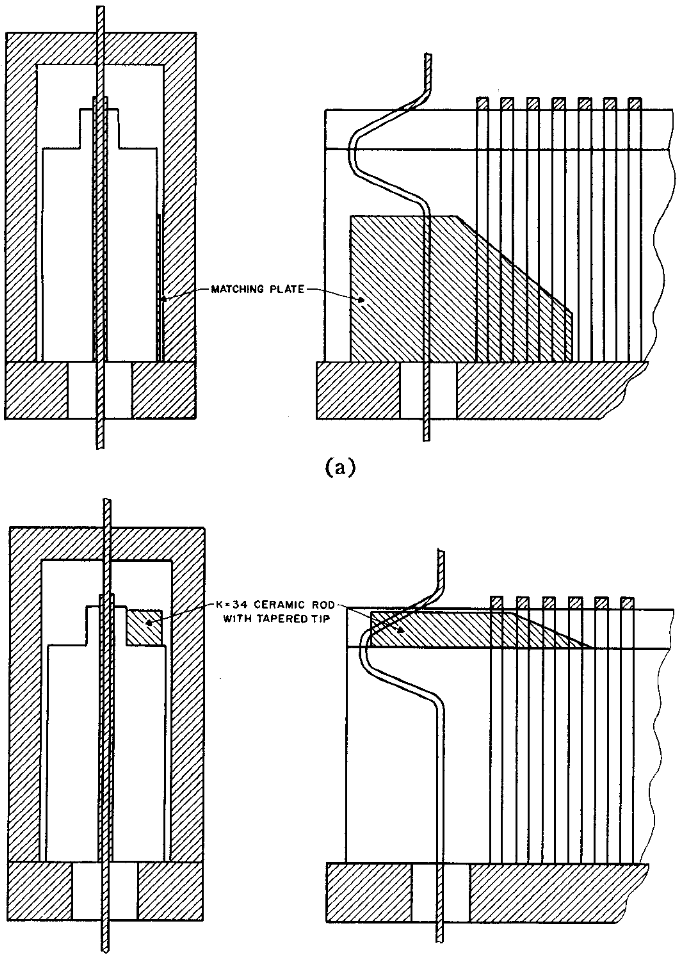

Fig. 4—Methods of lowering the lower cutoff frequency of the maser comb structure. (a) Metal or ceramic shim between dielectric loading and side wall. (b) Rod of high dielectric constant with tapered tip.

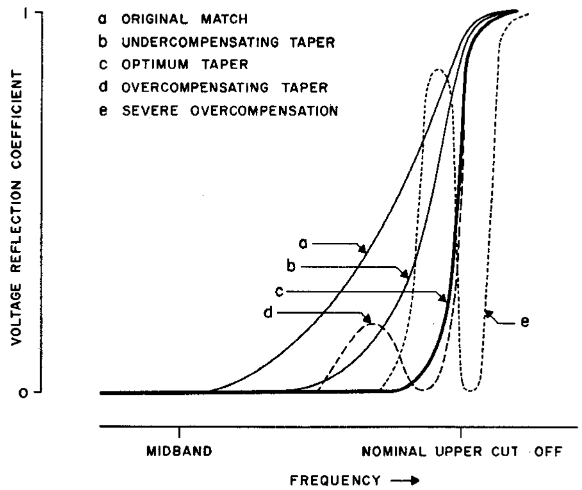

Fig. 5—Typical reflectometer tracings for the upper half of the pass band obtained with varying degrees of frequency widening in the matching taper.

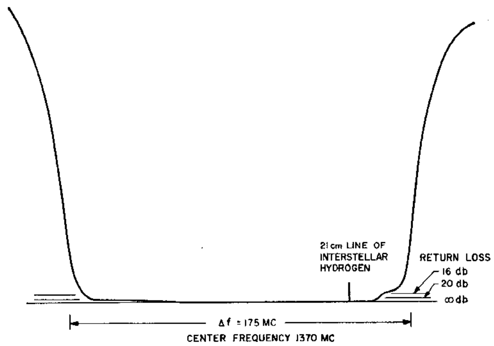

Fig. 6—Reflectometer tracing of typical comb structure to coaxial cable match using the frequency tapering technique described.

spite of differential thermal contraction and expansion between ruby and the copper comb. The optimum spacings between finger tips were determined experimentally [see Fig. 3(c)] and they are realized with close tolerances by inserting an appropriately cut ceramic finger spacer. The changes in the finger spacings are hardly visible to the unaided eye and they may, at the outermost finger, increase the upper cutoff frequency by about 50 Mc.

At the lower cutoff frequency, the electrical field extends from the comb to the surrounding waveguide housing. This frequency was lowered in the experiments in three similar ways.

1) A tapered piece of copper sheet is inserted into the narrow gap between the ruby loading and the housing as shown in Fig. 4(a).
2) A ceramic material (alumina) of the same geometry may also be used.
3) A ceramic rod of high dielectric constant ($K=34$) with a square cross section and tapered at the tip is laid into a step in the dielectric loading as shown in Fig. 4(b).

All three methods work satisfactorily. The third one is somewhat preferable because of less critical dimensions.

The empirical procedure for establishing an optimum taper may be discussed with the aid of Fig. 5. It shows typical reflectometer tracings for the upper half of the pass band as they appear during the matching procedure. Curves *a* through *e* are obtained with increasing tapering of the pass band by progressively widening the gaps between the first few comb fingers [Fig. 3(c)]. Curve *a* shows the original match, *b* is typical of an undercompensated match and *c* shows the optimum broad banding; the extra hump in *d* indicates overcompensation and with even greater spacings a sharp resonance is distinguished as in *e*. Corresponding statements apply to the lower half of the pass band where the reflectometer tracings would be a mirror image of those shown in Fig. 5. The effectiveness of possible bandwidening schemes for matching purposes is best investigated by a series of reflectometer tracings and comparing them with those shown in Fig. 5.

In the matching work with the comb structure, the probe shape and spacing was first adjusted to give a good match at midband. Then the optimum spacings between the comb fingers in the taper were determined using controlled finger bending under a microscope. After insertion of a ceramic spacer comb machined to these optimum dimensions it was necessary to readjust the probe spacing to recover the good match at midband. Finally the ceramic rod with tapered tip was inserted until the reflection diagram for the lower half of the pass band assumed the optimum, nearly square shape. The ceramic rod was then epoxy bonded in place. A tracing obtained after this procedure is reproduced in Fig. 6. It indicates a return loss of 20 db (VSWR = 1.2) or better over 90 per cent of the 175 Mc pass band. The edges of the pass band here were identified with points of about 100 db transmission loss. The frequency widening at the first comb finger amounts to about ± 50 Mc. This matching performance can be achieved with any comb using the same finger spacer and following the steps mentioned provided its mechanical deviations from true periodicity, *i.e.*, the machining tolerances, are sufficiently small. Difficulties in matching could always be traced to this cause. So far it was not necessary to consider the shape of the taper in detail. The ceramic rod controlling the lower cutoff frequency has a linear taper which causes most likely a linear taper in that frequency. The finger spacer features a geometric progression in the gap widths resulting probably in a similar progression of the upper cutoff frequencies. Both types apparently produce equally satisfactory matches.

The appearance of a well-resolved resonance as a consequence of excessive structure widening (see curve *e* in Fig. 5) leads to an interesting observation. The resonance condition at frequency f is approximately

$$\sum_n \theta_n(f) = \pi$$

for a single, resolved resonance near the lower cutoff frequency and

$$\sum_n [\pi - \theta_n(f)] = \pi$$

for a resonance near the upper cutoff. Here θ_n is the phase shift between filter element $n-1$ and n and f is a frequency contained in the transmission band of the widened taper section, but not in the regular structure bandwidth. The sum is taken along a line typified by the dotted lines in Fig. 2(a) and 2(c). In the matching technique, however, resonances are avoided. This means that in actual matching tapers, the indicated sums are always smaller than π. This suggests two interesting conclusions. One is that the amount of tapering permissible is limited with respect to both the number N of filter elements involved and the frequency widening. More frequency width can be employed in a design if the number of affected filter elements is less, and vice versa. The other observation is that, outside the regular pass band, the taper section is reactive and hence leads to perfect reflection outside the regular structure pass band. The small loss present in reality alters this only insignificantly.

In conclusion it may be said that the present approach offers a systematic way of deriving experimentally a design for a matching structure into iterated filter structures. The matches obtained with the maser comb structure are far superior to those previously attainable. This approach is equally applicable to other filter structures, in particular those used in high power electron beam tubes. An example is the transmitter tube for the Telstar ground station[8] where, however, the need for widening the outermost filter elements was derived by different theoretical arguments.

Acknowledgment

The authors acknowledge encouragement from E. D. Reed, H. E. D. Scovil and D. A. Chisholm.

[8] R. J. Collier, G. D. Helm, J. P. Laico and K. M. Striny, "The ground station high-power traveling-wave tube," *Bell Syst. Tech. J.*, vol. 42, pp. 1829–1861; July, 1963.